Goal

$$0 \leq Df(\mathbf{x}) \quad \boxed{\forall \mathbf{x} \in \mathcal{K}}$$

Examples for D:

non-negativity	$0 \leq f(x)$
monotonicity	$0 \leq f'(x)$
convexity	$0 \leq f''(x)$
v-monotonicity	$0 \leq \partial^{\mathbf{e}_j} f(\mathbf{x}) \, \forall j$ or $0 \leq \partial^{\mathbf{e}_d} f(\mathbf{x}) \leq \ldots \leq \partial^{\mathbf{e}_1} f(\mathbf{x})$
supermodularity	$0 \leq \frac{\partial^2 f(\mathbf{x})}{\partial x_i \partial x_j} \; \forall i \neq j \in [d]$

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- Rich fn classes: $f \in \mathcal{H}_k \xrightarrow{\text{spec}}$ Fourier analysis, polynomials, splines, ...

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Tightening [Aubin-Frankowski and Szabó, 2022]: consistent

original constraint:	SOC constraint:
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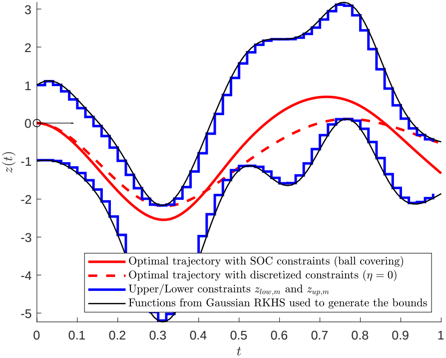

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