Kernel Regression with Hard Shape Constraints

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Shape-constrained regression problems [1, 2, 3] arise in a large number of applications. Economic theory dictates that utility functions are increasing and concave, demand functions of normal goods are downward sloping, production functions are concave or S-shaped, or that the link function in a single index model is typically monotone. In finance, European and American call option prices are convex and monotone in the underlying stock price and increasing in volatility, bond yield curves are monotone and concave in time to maturity, the conditional value-at-risk measure is increasing w.r.t. the significance level. In stochastic control and reinforcement learning the value function is regularly assumed to be convex.

Leveraging prior knowledge expressed in terms of shape structures has several advantages: the resulting techniques allow for estimation with smaller sample size, handle larger scale tasks, and help interpretability. Despite the numerous practical benefits the construction of shape-constrained estimators is quite challenging, existing techniques often tackle the shape requirements (i) in a soft fashion (without out-of-sample guarantees), (ii) by specialized transformation of the variables (such as logarithmic or translog specifications), or (iii) using of highly restricted functions classes such as polynomial splines.

In this work, we focus on the problem of shape-constrained regression with pointwise inequality constraints:

$$\min_{f\in\mathcal{F}_k} L\left((\mathbf{x}_n, y_n, f(\mathbf{x}_n))_{n\in[\![1,N]\!]}\right) + R\left(\|f\|_k\right), \text{ subject to } b_0 \leq D(f-f_0)(\mathbf{x})\ \mathbf{x}\in K_0, \tag{1}$$

where the hypothesis space is a reproducing kernel Hilbert space $\mathcal{F}_k$ of $\mathbb{R}^d \to \mathbb{R}$ functions; the samples $\{(\mathbf{x}_n, y_n)\}_{n\in[\![1,N]\!]}$, the regularization function R, the differential operator D, the constant b_0, the function f_0, and the compact nondiscrete set K_0 are given.

We show how second-order cone programming techniques can be applied to solve a strengthened version of (1), and hence to satisfy *strictly* the imposed shape constraints. In addition, we provide performance guarantees w.r.t. to the solution of the original problem (1). We demonstrate the efficiency of the proposed technique in joint quantile regression and in the context of transportation systems.

References

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